Exact answer is an integral of $v(t)$ from $t_i \to t_f$:

$$\Delta x = \int_{t_i}^{t_f} v(t)\,dt$$

$$x_f - x_i = \int_{t_i}^{t_f} v(t)\,dt$$

$$x_f = x_i + \int_{t_i}^{t_f} v(t)\,dt.$$

or

$$x(t_f) = x(t_i) + \int_{t_i}^{t_f} v(t)\,dt$$

Sometimes we call $t_i = t_0$ and $t_f = t$

$$x(t) = x(t_0) + \int_{t_0}^{t} v(t')\,dt'$$

The "prime" is meant to "dress up" the letter to make it different from the limit.

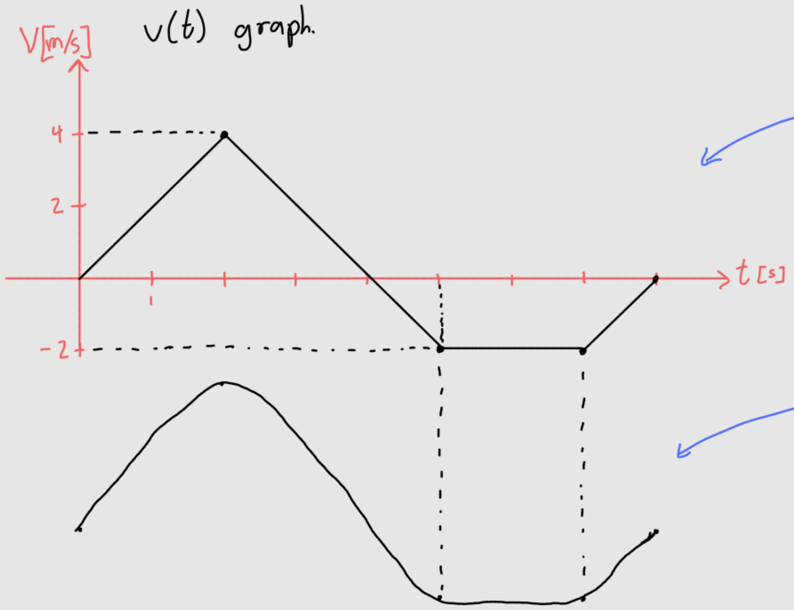
$$x \underset{\int dt}{\overset{d/dt}{\rightleftarrows}} v \underset{\int dt}{\overset{d/dt}{\rightleftarrows}} a$$

x and v: $\quad v = \frac{dx}{dt} \quad$ OR $\quad x(t) = x_0 + \int_{t_0}^{t} v(t')\,dt'$

v and a: $\quad a = \frac{dv}{dt} \quad$ OR $\quad v(t) = v_0 + \int_{t_0}^{t} a(t')\,dt'$

These are effectively definitions.

Ex. 1 A particle, constrained to move on the x-axis has the following $v(t)$ graph.



much easier to differentiate and integrate even though it's more impractical.

horrible to integrate and differentiate, even though it's likely more realistic.

(a) Determine the particle's position change from $0 \to 4s$.

$$\Delta x_{0\to 4s} = \left[\begin{array}{c}\text{area under } v(t)\\ \text{from } 0\to 4s\end{array}\right]$$
$$= \frac{1}{2}(4s)(+4 m/s)$$
$$= +8 m$$

North of where it started.

(b) Determine its position change from $4s \to 8s$.

$$\Delta x_{4s\to 8s} = \left[\begin{array}{c}\text{area under } v(t)\\ \text{from } 4s\to 8s\end{array}\right]$$
$$= \frac{1}{2}(1s)(-2 m/s) + (2s)(-2 m/s) + \frac{1}{2}(1s)(-2 m/s)$$
$$= -6m$$

south of where it was.

(c) Determine its position change from $0 \to 8s$.

$$\Delta x_{0\to 8s} = \Delta x_{0\to 4s} + \Delta x_{4s\to 8s}$$
$$= +8m - 6m = +2m$$

North of where you started at 0s when at 8s.

(d) Determine the distance traveled by this particle from $0 \to 8s$.

Distance, for example, tracks the "number of steps".

$$D_{0\to 8s} = 8m + 6m = 14m$$

If you consistently move in one direction, the position change's value is identical to the distance; however, if at some point you end up moving opposite, then the position change's value is <u>not</u> the same as the distance traveled.

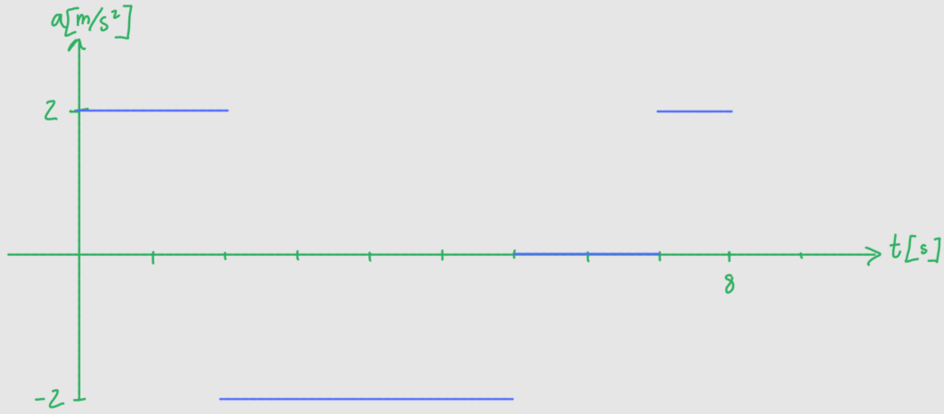
(e) Draw an accurate graph for this particle's acceleration from $0 \to 8s$.

$0 \to 2s$: $\quad a = \dfrac{4\,m/s - 0}{2s - 0} = +2\,m/s^2$

$2s \to 5s$: $\quad a = \dfrac{-2\,m/s - 4\,m/s}{5s - 2s} = -2\,m/s^2$

$5s \to 7s$: $\quad a = 0$

$7s \to 8s$: $\quad a = +2\,m/s^2$



Ex. 2 The velocity of a particle as a function of time for $t \in [0, \infty)$ is given as:

$$v(t) = 3\alpha t^2,$$

where α is a positive constant; namely, $\alpha = 4.0\,m/s^3$

It's known that this particle is 10m North of the origin initially, with "North" taken as "+".

(a) Determine $x(t)$ for any $t \geq 0$.

$x(t=t_0) = x(t=0) = +10\,m$

$\Rightarrow v(t') = 3\alpha t'^2$

$$x(t) = \boxed{x_0} + \int_{t_0}^{t} v(t')\,dt'$$

$$x(t) = x_0 + \int_0^t 3\alpha t'^2\,dt'$$

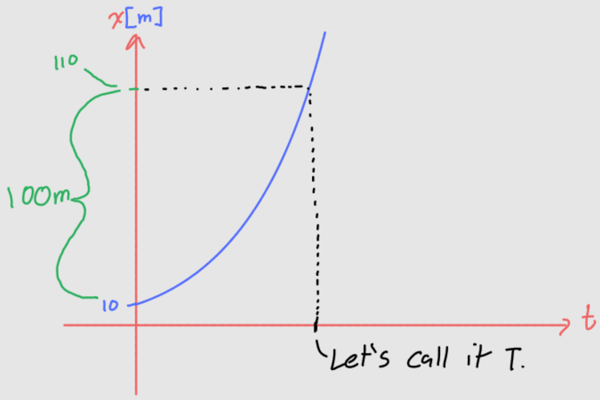
$$= x_0 + 3\alpha \int_0^t t'^2\,dt'$$

$$= x_0 + 3\alpha \left[\frac{t'^3}{3}\right]_{t'=0}^{t'=t}$$

$$= x_0 + 3\alpha\left(\frac{t^3}{3}\right)$$

$$\Rightarrow \boxed{x(t) = x_0 + \alpha t^3}$$

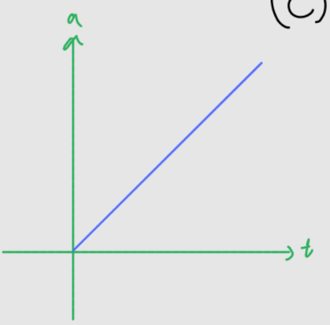
(b) Determine the amount of time elapsed from the beginning when this particle has traveled a distance of 100 m.



$$110\,m = x(T) = x_0 + \alpha T^3 \quad \left(\begin{array}{l}\text{where } \alpha = 4.0\,m/s^3 \\ \text{and } x_0 = +10\,m\end{array}\right)$$

$$x(T) - x_0 = \alpha T^3$$
$$\Rightarrow T^3 = \frac{x(T) - x_0}{\alpha}$$
$$\Rightarrow T = \left[\frac{x(T) - x_0}{\alpha}\right]^{1/3}$$
$$= \left[\frac{100m}{4.0m/s^3}\right]^{1/3}$$
$$= (25s^3)^{1/3}$$
$$= \sqrt[3]{25s^3} \simeq 3s$$

(C) Determine a(t) for $t \geq 0$.



$$a = \frac{d}{dt}(3\alpha t^2)$$
$$= 3\alpha \frac{d}{dt}(t^2)$$
$$= 3\alpha(2t) = 6\alpha t.$$