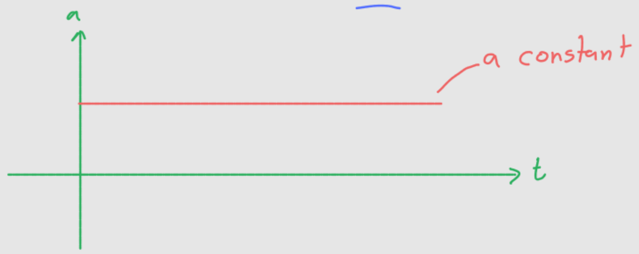
The simplest case of Non-Uniform Motion in 1D is that of motion with a constant acceleration.



We want to derive the so-called "equation of motion for constant acceleration in 1D"

$$\begin{cases} x(t) \\ v(t) \end{cases} \checkmark$$

Road map: Since a is known, to get $v(t)$ and $x(t)$, we have to integrate.

First: Let's determine $v(t)$.

$$v(t) = v_0 + \int_{t_0}^{t} a(t')\,dt'$$

$$= v_0 + \int_{t_0}^{t} a\,dt'$$

$$= v_0 + a\int_{t_0}^{t} dt'$$

$$= v_0 + a\big[t'\big]_{t_0}^{t}$$

$$v(t) = v_0 + a(t - \boxed{t_0})$$

not a variable

Second: Let's determine $x(t)$.

$$x(t) = x_0 + \int_{t_0}^{t} v(t')\,dt'$$

$$\Rightarrow v(t') = v_0 + a(t' - \boxed{t_0})$$

no adjustment.

$$x(t) = x_0 + \int_{t_0}^{t}\big[v_0 + a(t' + t_0)\big]dt'$$

$$= x_0 + \int_{t_0}^{t} v_0\,dt' + \int_{t_0}^{t} a(t' + t_0)\,dt'$$

$$= x_0 + v_0\int_{t_0}^{t} dt' + a\int_{t_0}^{t} t'\,dt' + a t_0 \int_{t_0}^{t} dt'$$

$$x(t) = x_0 + v_0(t - t_0) + a\left[\frac{t'^2}{2}\right]_{t_0}^{t} - a t_0 (t - t_0)$$

$$x(t) = x_0 + v_0(t - t_0) + \left[\frac{1}{2}a\underbrace{\boxed{(t^2 - t_0^2)}}_{(t + t_0)(t - t_0)} - a t_0 (t - t_0)\right]$$

$$x(t) = x_0 + v_0(t - t_0) + a(t - t_0)\left[\underbrace{\tfrac{1}{2}(t + t_0) - t_0}\right]$$

$$\tfrac{1}{2}t + \underbrace{\tfrac{1}{2}t_0 - t_0}_{-\frac{1}{2}t_0} = \tfrac{1}{2}(t - t_0)$$

$$\therefore x(t) = x_0 + v_0(t - t_0) + a(t - t_0)\left[\tfrac{1}{2}(t - t_0)\right]$$

$$\Rightarrow \boxed{x(t) = x_0 + v_0(t - t_0) + \tfrac{1}{2}a(t - t_0)^2}$$

Now, here are the equations of motion for constant acceleration in 1 D:

$$\begin{cases} x(t) = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2 \\ v(t) = v_0 + a(t - t_0) \end{cases}$$

These two equations tell us everything we need to know about ANY constant acceleration problem in 1 D.

To make solving problems of this type more efficient, let's use the equations of motion to eliminate each of the three common parameters between them to get three new equations that are dependent on the equations of motion.

First eliminate $(t - t_0)$

$$v = v_0 + a(t - t_0)$$

$$\Rightarrow v - v_0 = a(t - t_0) \Rightarrow t - t_0 = \frac{v - v_0}{a}$$

Plug into the "x" equation

$$x = x_0 + v_0(t - t_0) + \tfrac{1}{2}a(t - t_0)^2$$

$$= x_0 + \left[v_0 + \tfrac{1}{2}a(t - t_0)\right](t - t_0)$$

$$x - x_0 = \left[v_0 + \tfrac{1}{2}a\left(\frac{v - v_0}{a}\right)\right]\left(\frac{v - v_0}{a}\right)$$

$$\Rightarrow x - x_0 = \left[v_0 + \tfrac{1}{2}v - \tfrac{1}{2}v_0\right]\left(\frac{v - v_0}{a}\right)$$

$$= \frac{1}{2}\,\frac{(v + v_0)(v - v_0)}{a}$$

$$\boxed{= \frac{1}{2}\,\frac{(v^2 + v_0^2)}{a}}$$

Multiply by $2a$: $2a(x - x_0) = v^2 - v_0^2$

$$\therefore \boxed{v^2(x) = v_0^2 + 2a(x - x_0)}$$

Second, eliminate a

$$v = v_0 + a(t - t_0) \Rightarrow a = \frac{v - v_0}{t - t_0}$$

$$x = x_0 + \left[v_0 + \tfrac{1}{2}a(t - t_0)\right](t - t_0)$$

$$x - x_0 = \left[v_0 + \tfrac{1}{2}\left(\frac{v - v_0}{\cancel{t - t_0}}\right)\cancel{(t - t_0)}\right](t - t_0)$$

$$x - x_0 = \left[v_0 + \tfrac{1}{2}v - \tfrac{1}{2}v_0\right](t - t_0)$$

$$\Rightarrow \frac{x - x_0}{(t - t_0)} = \frac{\tfrac{1}{2}(v - v_0)(t - t_0)}{(t - t_0)}$$

$$\boxed{\frac{x - x_0}{t - t_0} = \frac{1}{2}(v + v_0)}$$

1D motion with constant acceleration

① $x(t) = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$

② $v(t) = v_0 + a(t - t_0)$

③ $v^2(x) = v_0^2 + 2a(x - x_0)$

④ $\frac{x - x_0}{t - t_0} = \frac{1}{2}(v + v_0)$

⑤ $x(t) = x_0 + v(t - t_0) - \frac{1}{2}a(t - t_0)^2$

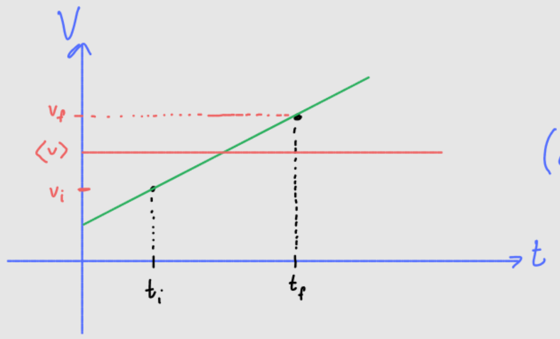
- 1 and 2 are the "equations of motion" for constant acceleration in 1D
- 3 was obtained by eliminating $(t - t_0)$ from 1 and 2
- 4 was obtained by eliminating a from 1 and 2
- 5 will be obtained by eliminating v_0 from 1 and 2

(4) $\boxed{\frac{x - x_0}{t - t_0}} = \frac{1}{2}(v + v_0)$ Can have an implicit sign.

this is the average velocity $\langle a \rangle$

For a constant acceleration problem, you can determine the average velocity in same time interval by literally averaging the initial and final instantaneous velocities in that interval.

$$\langle f \rangle \equiv \frac{1}{b-a}\int_a^b f(x)\,dx$$



(constant acceleration)

$$v = v_0 + a(t - t_0)$$

$$\Rightarrow v_0 = v - a(t - t_0)$$

$$x - x_0 = \left[v_0 + \frac{1}{2}a(t - t_0)\right](t - t_0)$$

$$x - x_0 = \left[v - \frac{2}{2}a(t - t_0) + \frac{1}{2}a(t - t_0)\right](t - t_0)$$

$$= \left[v - \frac{1}{2}a(t - t_0)\right](t - t_0)$$

$$x - x_0 = v(t - t_0) - \frac{1}{2}a(t - t_0)^2$$

$$\boxed{x(t) = x_0 + v(t - t_0) - \frac{1}{2}a(t - t_0)^2}$$

$v_0 = 0$

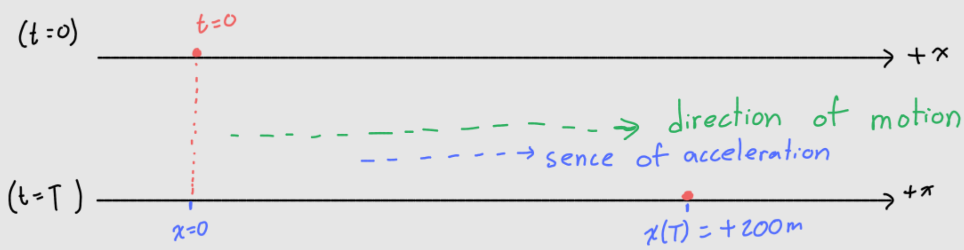
Ex. 1 A car starts from rest and constantly accelerates at $1.00\,m/s^2$ indefinitely.

(a) How much time does it take for the car to have traveled 200m from when it began accelerating.

Know: $\boxed{v_0}$, a, $\boxed{x}$

0 $\quad$ $200m$

Want: $t - t_0$ or just t if $t_0 = 0$



Equation 1: $\quad x(T) = 0 + 0 + \frac{1}{2}a(T - 0)^2$

$$\Rightarrow T^2 = \frac{2x(T)}{a}$$

$$\Rightarrow T_{\pm} = \pm\sqrt{\frac{2x(T)}{a}} \Rightarrow T = \sqrt{\frac{2x(T)}{a}} = 20.0s$$

(b) What is the car's speed at the moment found in (a)?

What if (a) and (b) were swapped?

$\Rightarrow$ The most efficient solution is using Eq. 3.

$v^2 = 0 + 2a(x - 0) \Rightarrow v = \sqrt{2ax}$

$\Rightarrow v_{\pm} = \pm\sqrt{2ax} \qquad = 20.0\,\text{m/s}$

One prominent example of constant acceleration movement is that of "freefall."

"Free Fall" is a type of motion in which the object that's moving is ONLY affected by the planet's gravitational influence.

Near Earth's surface, which is where we're most interested, there's a problem: namely, that's air.

Two ways to meet the free fall criteria:

(1) Set up a vacuum chamber.

(A bit impractical on a large scale)

(2) The air drag is kept to a minimum.

- * Keep the speed to be small relative to the air.
- * Use an aerodynamic object.

Near Earth's surface, <u>any</u> object that is in free fall accelerates at exactly the same rate and direction:

$g = 9.80\,\text{m/s}^2$, <u>down</u>

type of acceleration (gravitational acceleration)