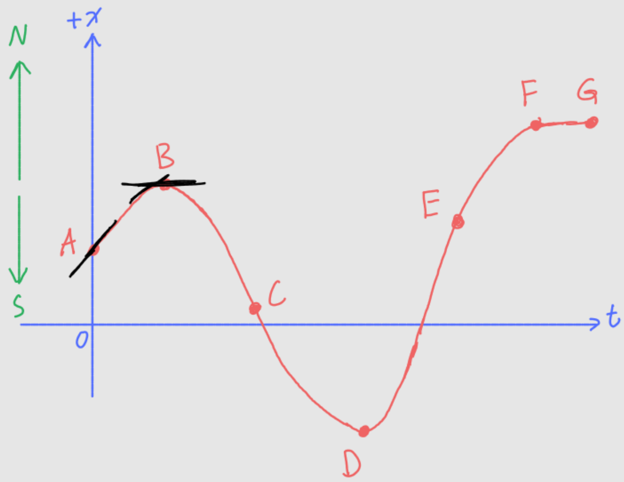
Graphical Examples:

Qualitative discussion.



For each succesive pair of points, comment on how the particle's motion is behaving.

A → B: Particle starts off fast heading N, then consistently slows down while heading N until it stops momentarily at B.

$V > 0$ (b/c slope is positive)

$a < 0$ (b/c C.D.)

B → C: From rest, it speeds up while going South.

$V < 0$

$a < 0$

C → D: Slows down to a momentary stop while going South.

$V < 0$

$a > 0$

D → E: From rest, speeds up while heading N.

$V > 0$

$a > 0$

E → F: "Same" as A → B

F → G: As it came to rest at F from E, it remains at rest throughout.

$V = 0$

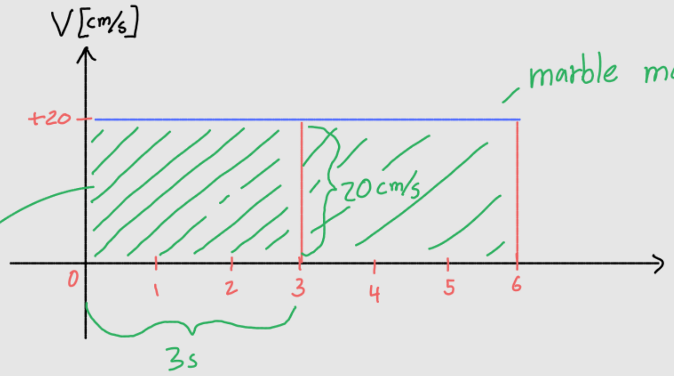
$a = 0$

$$X \underset{}{\overset{d/dt}{\rightleftarrows}} V \overset{d/dt}{\rightleftarrows} a$$

this is also possible

Let's start off with a graph of $v(t)$ in $\boxed{\text{uniform motion}}$.

Also known as: motion with constant velocity ↵



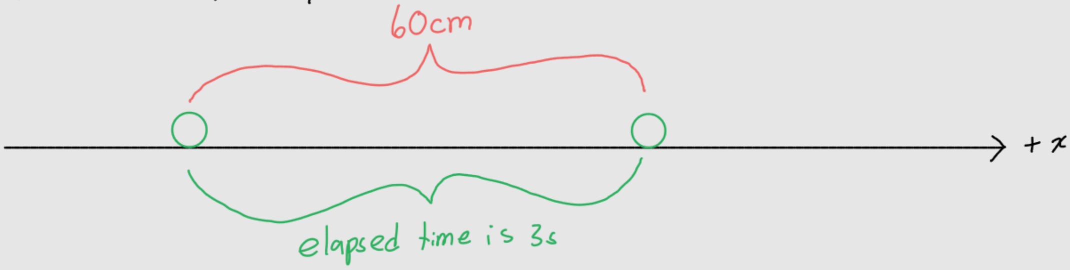
From 0 → 3s, how much has the marble advanced?

The area under this graph is a representation of the answer to this question.

$(+20\,cm/s)(3s) = +60cm$

What about 3s → 6s? +60cm

Certainly the marble advances a certain amount North, but we can't really say what its absolute position is.



This area under the curve of $v(t)$ only gives the change in position, not the position itself.

Uniform motion: $\Delta x = v\Delta t$

$\Delta x = x - x_0$

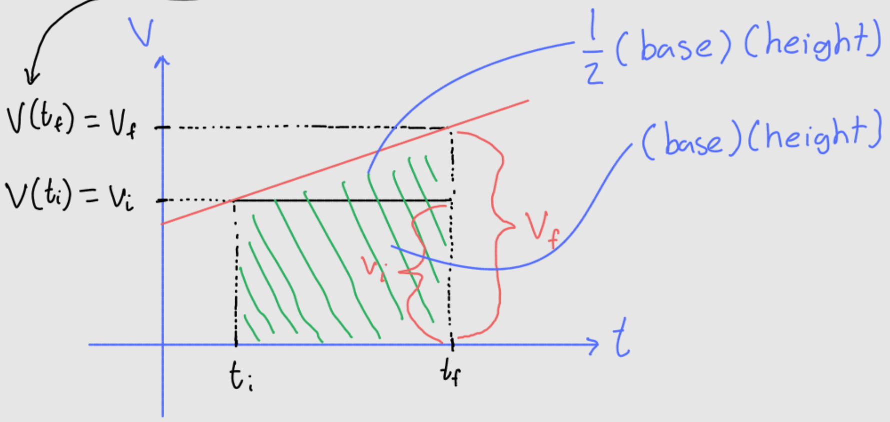
$$\boxed{x(t)} = x_0 + \underbrace{\boxed{v(t-t_0)}}_{\text{area}}$$

To get $x(t)$, we not only need the area the area under $v(t)$ graph, but also x_0.

In non-uniform motion, nothing changes qualitatively:

$$\Delta x = \underbrace{\begin{pmatrix} \text{area under the } v(t) \text{ graph} \\ \text{for } t_i \to t_f \end{pmatrix}}_{\text{quantitatively, this is annoying!}}$$

A specific case where it's not so annoying is when $v(t)$ is a straight line.

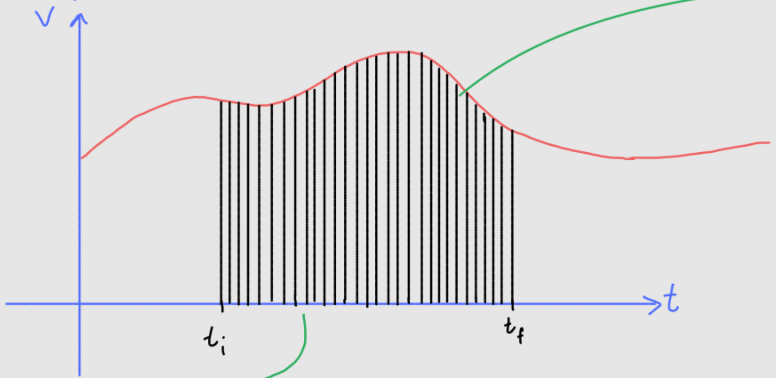


$$\Delta x = (\text{area of rectangle}) + (\text{area of triangle})$$

$$= v_i \Delta t + \frac{1}{2}\left[\frac{(v_f - v_i)}{\Delta t}\right](\Delta t)^2$$

$$= \underbrace{v_i \Delta t}_{\text{a bit of foreshadowing}} + \frac{1}{2}\underbrace{\boxed{a}}_{\text{constant acceleration!}}(\Delta t)^2$$

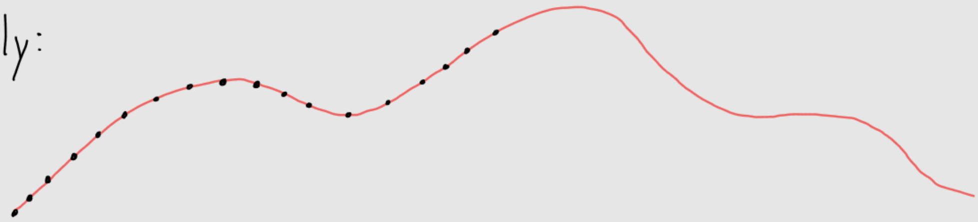
Now, what about other cases?



$\underbrace{\text{eventually you populate with infinite number of rectangles, and then sum up the area of each rectangle}}_{\text{Riemann Sum}}$

We can figure out the area approximately if we "pixelate" the area.

Physically:



everything is discrete $\Rightarrow$ have to do numerical sum.

You'll get an exact answer as your sampling tends to ∞; however, that might be meaningless given your instrumentation's precision.