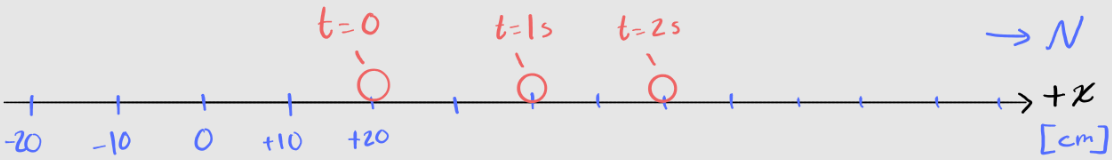
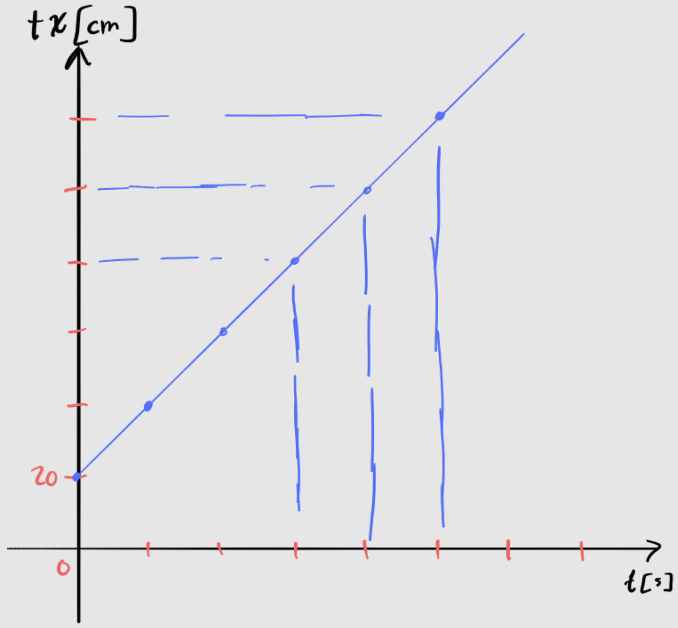
II: The marble is moving North in uniform motion.



One photo is not enough to establish movement. You require one more photo to do so.

In uniform motion, the spacing between succesive, equally timed photos is the same.

t[s]	x[cm]
0	+20
1	+40
2	+60
3	+80
4	+100
5	+120
⋮	⋮



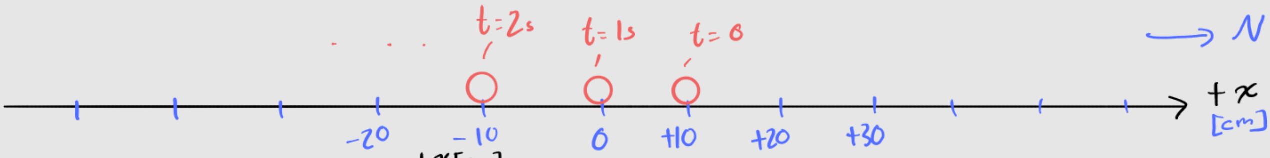
I and II are similar in the sense that they have x vs. t graphs that are straight; however, their slopes are different.

I: slope $= \frac{\text{rise}}{\text{run}} = \frac{0}{\text{not zero}} = 0$

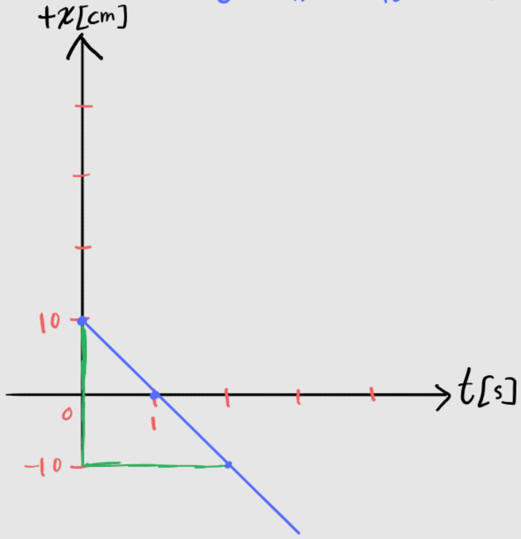
II slope $= \frac{\text{rise}}{\text{run}} = \frac{+80\text{cm}}{4\text{s}} = +20\text{cm/s}$

Technically, this slope is a $\boxed{\text{VELOCITY}}$, NOT a speed.

III: The marble is moving South in uniform motion.



t[s]	x[cm]
0	+10
1	0
2	-10
3	-20
4	-30
⋮	⋮



III: slope $= \frac{-20\,cm}{2s} = -\underbrace{\boxed{10\,cm/s}}$

Marble III is moving $\boxed{10cm/s}$, southward

Speed tells you how fast an object is moving.
Velocity tells you not only the speed, but also the direction of movement

For now, we'll label velocity as: V
As such, we'll label speed as: |V|

the absolute value of the velocity

We can now put these quantities (t, x, v) together for the case of uniform motion.

Because V is the slope of the x-vs-t graph, and we know the graph is a straight line, then we can write:

position change

$$V = \frac{x_f - x_i}{t_f - t_i}$$

i = initial
f = final

elapsed time

$$\therefore V = \frac{\Delta x}{\Delta t}$$ → velocity from position and time info

↕

$$\Delta x = V \Delta t$$ → position from velocity and time info

↕

$$\Delta t = \frac{\Delta x}{V}.$$

Alternatively, it's common to see the following notation:

t_0 : initial time
x_0 : initial position
t : final time
x : final position

$$x - x_0 = v(t - t_0)$$

$$\Rightarrow \quad x = x_0 + v(t - t_0)$$

$$\Rightarrow \boxed{x(t) = x_0 + v(t - t_0)}$$

The Equation of motion for Uniform Motion

Given t_0, x_0, V, and t, we can predict x.

In math:

$$y - y_1 = m(x - x_1)$$

$$y = y_1 + m(x - x_1)$$

Non-Uniform Motion (1D)

In uniform motion, we saw that the velocity was constant.

speed
$\boxed{\text{direction of motion}}$

"Uniform motion in 1 D" is redundant.

When we say "non-uniform motion", 3 possibilities:

Neither the speed nor the direction of movement are constant
↓
at least, you're moving in 2D

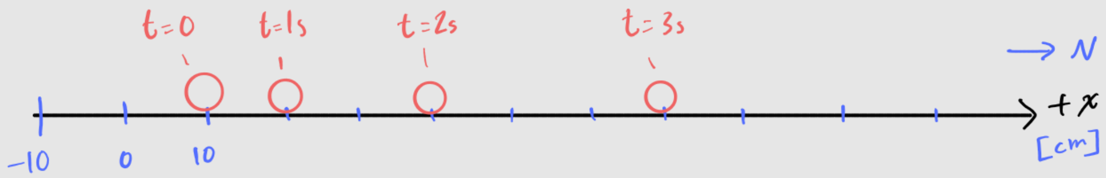
the speed is constant but the direction of movement isn't.
↓
at minimum, moving in a plane

the direction of movement is constant, but the speed isn't
↓
this is our focus for 1D

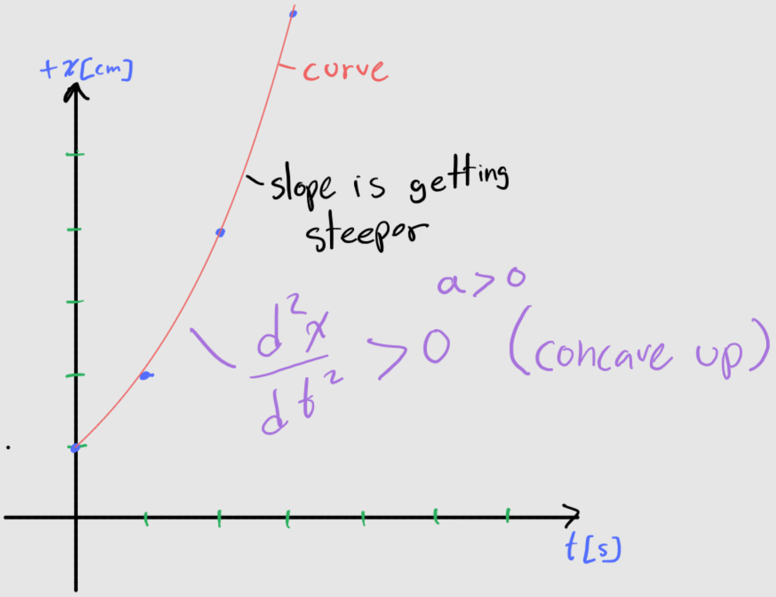
There's only two possibilities of non-uniform motion in 1D:

* increasing speed ("speeding up")
* decreasing speed ("slowing down")

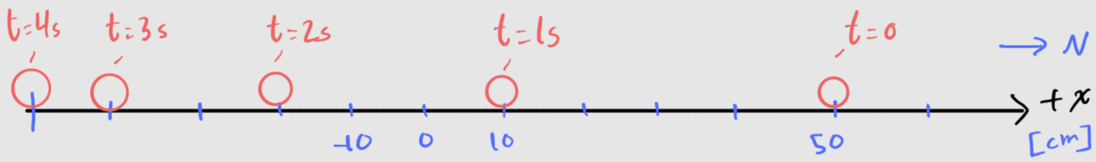
A: Marble is speeding up as it goes North.



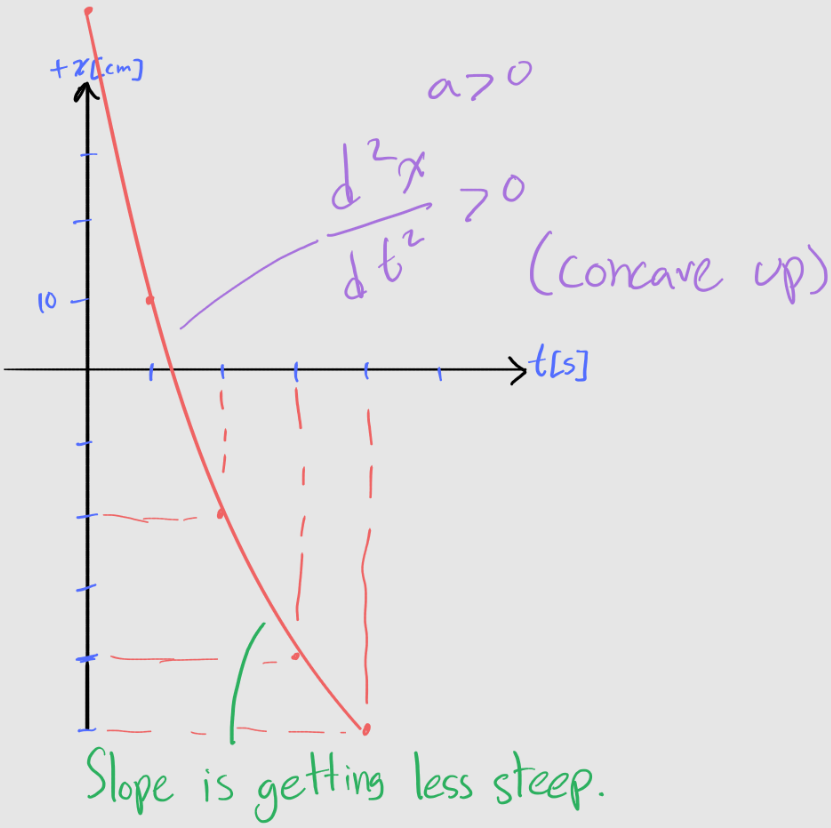
t[s]	x[cm]
0	+10
1	+20
2	+40
3	+70



B: Marble is slowing down as it goes South.



t[s]	x[cm]
0	+50
1	+10
2	-20
3	-40
4	-50

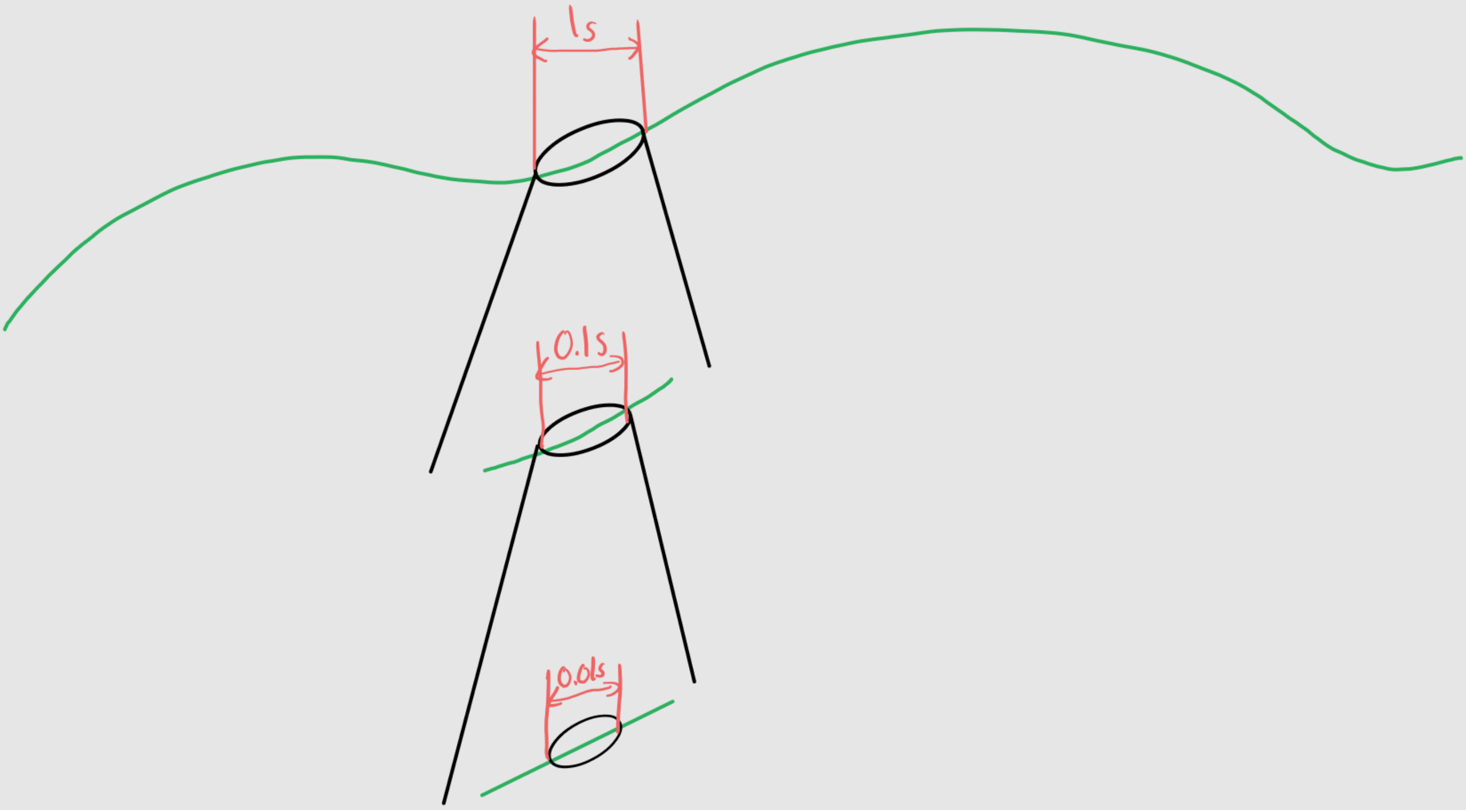


A nice fact:

Anything curved looks straight LOCALLY!

The more you "zoom-in" to a curve the straighter it'll look in that region.

for an x-vs-t graph, this is achieved by taking a smaller, and smaller, and smaller time interval.



Generally, how much shrinking needs to be done until the curve is straight?

↳ It depends on the system.

* For typical macro objects in everyday life, a "blink of an eye" is enough. $\approx 0.2s$
* For an atomic system it's not uncommon for processes to last less than one nanosecond (0.000000001s)
* For a geological process, maybe one year will suffice.

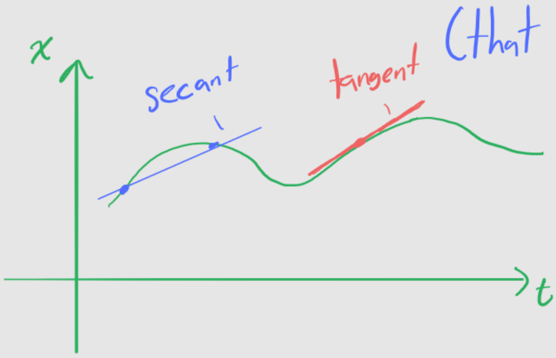
Under these circumstances, when the time interval is shortened enough to have x-vs-t in that window be approximately straight, the slope becomes what we call the "instantaneous velocity"

"instant of time" is a point on the time axis.

Practically, an instant of time is really no different than a very, very short interval of time.

$$\text{Instantaneous Velocity} = \frac{(\text{Position change in a very very short time interval})}{(\text{that time interval})}$$

$$\text{Average Velocity} = \frac{(\text{position change in a not-so-short time interval})}{(\text{that time interval})}$$



Formally:

$$\text{inst. vel.:} \quad v \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}.$$

$$\text{ave. vel.:} \quad \langle v \rangle \equiv \frac{\Delta x}{\Delta t}$$

Refer to marble A and B in purple

$\cancel{\bar{v}}$

$\cancel{v_{ave}}$

Non - Uniform Motion

Instantaneous Velocity: $V \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \equiv \frac{dx}{dt}$

Average Velocity: $\langle v \rangle = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$

$$= \frac{x - x_0}{t - t_0}$$

Instantaneous Acceleration: $a \equiv \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} \equiv \frac{dv}{dt}$

Average Acceleration: $\langle a \rangle = \frac{\Delta V}{\Delta t}$

In x-vs-t graph: The velocity is the slope. $\frac{dx}{dt}$

The concavity is the acceleration. $\frac{d^2x}{dt^2}$

Velocity is a quality that has a size and a direction.

speed — direction of movement.

Acceleration also has a size and direction.

magnitude of the acceleration

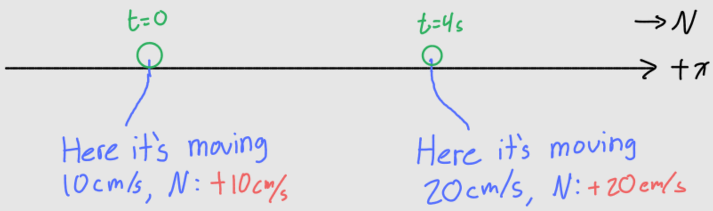
As far as the direction of any of these quantities is concerned, once we adopt a coordinate system, the signature carries the direction.

\+ −

For example, if "North" is "$+x$", then a marble moving North has a positive velocity, while a marble accelerating South has a negative acceleration.

Some quick computational examples

1)

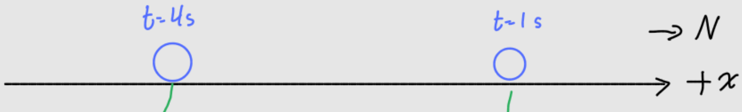


$$\langle a \rangle = \frac{\Delta V}{\Delta t} = \frac{V_f - V_i}{t_f - t_i} = \frac{+20\,cm/s - (+10\,cm/s)}{4s - 0}$$

$$= \frac{20 - 10}{4}\ cm/s^2$$

Northward $\rightarrow$ $= +2.5\ cm/s^2$

2)



Here it's moving 50cm/s, S.

⇒ −50 cm/s.

Here it's moving 20cm/s, S.

⇒ −20cm/s.

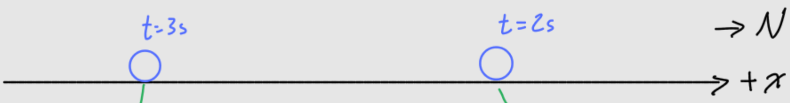
$$\langle a \rangle = \frac{v_f \boxed{-} v_i}{t_f - t_i} = \frac{(-50\ \text{cm/s}) \boxed{-} (-20\ \text{cm/s})}{4s - 1s}$$
$$= \left(\frac{-50+20}{3}\right) \text{cm/s}^2$$
$$= -10\ \text{cm/s}^2$$

points Southward

Yes, we do have a negative acceleration, but that sign doesn't imply anything about how the speed is changing!

In this example, it happens to be that the speed increases despite the acceleration being negative.

3)



Here the velocity is 15 cm/s, S

⇒ −15 cm/s

Here the velocity is 20 cm/s, S

⇒ −20cm/s

$$\langle a \rangle = \frac{(-15\ \text{cm/s}) - (-20\text{cm/s})}{3s - 2s}$$
$$= \left(\frac{-15+20}{1}\right) \text{cm/s}^2$$
$$= +5\ \text{cm/s}^2$$

Acceleration Points N.

The sign of Acceleration is <u>not</u> enough to establish how to establish how the velocity is changing. What matters is how acceleration's sign compares to the velocity's.

If they're the same sign ⇒ increasing speed.

If they're opposite signs ⇒ decreasing speed.