Error Function

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What is an Error Function?

The Error Function is defined as:

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, \mathrm{d}t \tag{1}$$

Graph of $\operatorname{erf}(x)$:

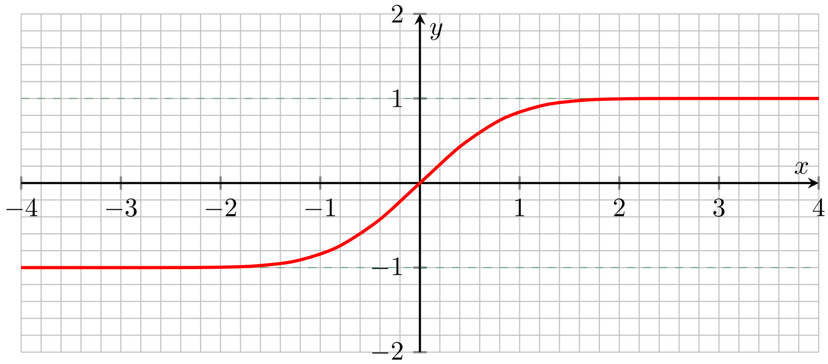

Figure 1: Graph of the Error Function $\operatorname{erf}(x)$.

Now you can see, this is an odd function! So $\operatorname{erf}(-x) = -\operatorname{erf}(x)$ As shown in Figure 1, the graph also has horizontal asymptotes of $y = 1$ and $y = -1$.

There is also a complementary error function:

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2}\, \mathrm{d}t \tag{2}$$

There's an interesting property that arises when we combine these two error functions:

$$\operatorname{erf}(x) + \operatorname{erfc}(x) = 1 \tag{3}$$

Now, why would we care about this? We can't solve the integral but it appears frequently in statistics, heat transfer, and physics, so we gave it a function name and manually computed its values for later reference.

Now that you know what it is and how the graph looks, let's try some problems!

1 $\displaystyle\int_0^t x^2 e^{-x^2}\,\mathrm{d}x$

2 $\displaystyle\int \operatorname{erf}(x)\,\mathrm{d}x$

3 $\displaystyle\int_1^t \frac{1}{e^{x^2}\,\mathrm{erf}(x)}\,\mathrm{d}x$

4 $\displaystyle\int \frac{2e^{-x^2}}{\operatorname{erfc}(x) - \operatorname{erf}(x)}\,\mathrm{d}x$

5 $$\int \frac{2\ln\left(\operatorname{erf}(x)e^{-x^2}\right)\left(\frac{1}{\sqrt{\pi}}e^{-x^2} - x\cdot\operatorname{erf}(x)\right)}{\operatorname{erf}(x)}\,\mathrm{d}x$$